VIBRATION MECHANISMS IN THE SOLID PHASE OF A FLUIDIZATION BED

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The phenomenon of periodic vibrations in a fluidization bed is analyzed in the light of two mechanisms of interaction between solid phase particles. Equations are derived for the frequency, the wave velocity, and the wave vector of periodic-bed vibrations.

The authors consider here the vibration of solid phase particles in a fludization bed resulting from collective interaction and direct collisions between particles. A study and analysis of the motion of the solid phase in a fluidization bed reveals that individual particles and clusters of particles vibrate with different amplitudes and frequencies. Altogether, according to the experimental studies in [5] and [6], the vibration spectrum of solid phase particles is continuous and covers a wide range. As to the causes and nature of vibrations excited and sustained in the solid phase of a fluidization bed, they evidently cannot be attributed to any single mechanism but rather to an interplay between several different processes. Among the various vibratory processes occurring in a fluidization bed, one can single out a few types of processes as, for instance, the "shaking" small-amplitude high-frequency vibration of individual particles and vibrations resulting from an interference between particle clusters.

The interference between individual particles can result not only from a direct momentum transfer from one to another during collisions but also from a meshing of the hydrodynamic boundary layers which surround such particles [1]. At a sufficiently high stirring rate in the solid phase and high relative velocities of the phases, furthermore, the individually moving particles leave turbulent trails which can also interfere with one another as well as with the solid phase particles. The breakaway of turbulent vortices during a fast motion of solid phase particles can also result in a particle-vortex type interference. These interference modes belong to the near-range category, and the kinetics equation [2]

$$\frac{\partial f}{\partial t}+\mathbf{u}\nabla f+(g+\lambda_1 w^{\beta-1}\mathbf{w})\frac{\partial f}{\partial \mathbf{u}}-\frac{\partial}{\partial \mathbf{u}}(\lambda_2 \mathbf{u} f)-D\nabla_u^2 f-I=0 \tag{1}$$

will account for them in a statistical description of the solid phase motion. The solution to this equation is, to the first approximation,

$$f(\mathbf{r},\ \mathbf{u})=n(z)\left(\frac{m}{2\pi\theta}\right)^{3/2}\exp\left(-\frac{mu^2}{2\theta}\right), \tag{2}$$

where $\theta = mD/\lambda_2$. For an expanding fluidization bed, when $n \ll 6/\pi d^3$, function n(z) must satisfy the equation

$$R^3\frac{dn}{dz}=C(R^\beta-1)n, \tag{3}$$

where

$$R=\left(\frac{\lambda_1}{g}\right)^{1/\beta}w,\quad C=\left(\frac{\lambda_1}{g}\right)^{3/\beta}\frac{gw^3}{D}\lambda_2.$$

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The solution to Eq. (3) is the following function

$$n(z) = n_0 \exp\left[\frac{\lambda_2 g}{D}\left(1 - \frac{\lambda_1}{g} w^\beta\right) z\right]. \tag{4}$$

Function n(z), when expressed in this form, is an analog of the barometric formula for a fluidization bed. A formula like (4) was proposed in [3] and exponential factor was evaluated experimentally.

By extending the analogy with the kinetic theory of gases, as far as Eq. (1) is concerned, in the case of elastic collisions between particles one may expect that vibrations of the acoustic kind are propagated through the fluidization bed. The velocity of these acoustic waves is

$$v^* = \sqrt{\frac{2\theta}{m}} = \sqrt{\frac{2D}{\lambda_2}}. \tag{5}$$

Energy is transmitted here by sequences of elastic collisions between solid phase particles in the fluidization bed. The motion of individual particles in the bed should be vibratory with small amplitudes and high frequencies.

The quantity λ_2 can be determined from the expression

$$\lambda_2 = \frac{D}{g}\left(1 - \frac{\lambda_1}{g} w^\beta\right)^{-1} \frac{\ln n - \ln n_0}{z}. \tag{6}$$

For a bed of d = 2.5-4.0 mm particles fluidized with air at 2.5-5.0 m/sec velocity, the value of λ_2 is somewhere within 0.012-0.020 sec^{-1}. The diffusivity D under these conditions, in the velocity space, has been determined experimentally [5, 6] and found to range from 15 to 40 cm^2/sec^{-3}. Inserting these values into (5) yields the following estimate for the velocity of vibrations in such a fluidization bed:

$$v^* \simeq 0.4 - 0.8 \text{ m/sec}. \tag{7}$$

The frequency of vibrations is estimated from the relation

$$f^* \simeq \frac{v^*}{l}, \tag{8}$$

where l denotes the characteristic linear dimension of the fluidization bed. Under the given conditions f* ranges from 8 to 16 Hz.

We note that these estimates for v* and f* agree closely with the maximum velocities and vibration frequencies of individual particles observed in the experiments [5, 6] concerning the motion of the solid phase in a fluidization bed.

A consideration of far-range hydrodynamic interference forces in a fluidization bed will explain the mechanism by which collective vibrations of solid phase particles are brought about. The equation of the particle distribution function in the phase space of coordinates and velocities can be derived with the aid of the concept of a self-adaptive field [7]:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial r_i}\{f u_i\} + \frac{\partial}{\partial u_i}\{(a_{ik} u_k + b_i) f\} = 0, \tag{9}$$

where $a_{ik}u_k + b_i$ is the self-adapted hydrodynamic interference force. When $\theta = 0$ or when

$$f = n_0 \delta(\mathbf{u} - \langle \mathbf{u} \rangle) \tag{10}$$

the phase velocity of collective vibrations in a fludization bed is determined from the relation

$$\frac{\omega}{k} = \frac{k\langle u \rangle}{k(1 - A)}, \tag{11}$$

where

$$A = \frac{3}{8} n_0 d^3 \frac{\rho_f/\rho_r}{1 + \frac{1}{2}\rho_f/\rho_s}.$$

The generalization of (11) for states with $\theta \neq 0$ is given in [7].

The phase velocity of collective vibrations under given conditions can be estimated on the basis of relation (11), considering that the magnitude of A is then of the order of $3 \cdot 10^{-2}$. Formula (11) yields now the phase velocity of collective vibrations of the order of 3-8 cm/sec in a coordinate system referred to the center of inertia of the solid phase particles.

During the fluidization of dielectric particles in industrial apparatus, static charge accumulates which can become quite appreciable. The electric interference between particles has then a pronounced effect on the vibratory motion of solid phase particles. The kinetics equation of the distribution function which takes into account not only hydrodynamic but also electric interference will be

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial r_i}\{fu_i\}+\frac{\partial}{\partial u_i}\left\{f\left[(a_{ik}u_k+b_i)-\frac{q^2\int\frac{\mathbf{r}_2-\mathbf{r}}{|\mathbf{r}_2-\mathbf{r}|^3}f(\mathbf{r}_2,\mathbf{u}_2,t)\,d^3u_2d^3r_2}{m\left(1+\frac{1}{2}\rho_f/\rho_s\right)}\right]\right\}=0. \tag{12}$$

The method in [8] of analyzing vibrations in multiparticle systems yields, in the linear approximation, a relation between the frequency of collective vibrations and the wave vector. It is assumed here that vibrations occur near a state corresponding to a distribution function of the (10) kind. The coordinate system has been selected so that the fluid comes to a standstill at infinity. The corresponding dispersion equation is

$$\det\left\|\delta_{ij}-A\frac{\omega}{\omega-\mathbf{k}\langle\mathbf{u}\rangle}\cdot\frac{k_ik_j}{k^2}-\Omega^2\frac{k_ik_j}{(\omega-\mathbf{k}\langle\mathbf{u}\rangle)^2k^2}\right\|=0, \tag{13}$$

where

$$\Omega^2=\frac{4\pi q^2 n_0}{m(1+1/2\rho_f/\rho_s)}.$$

When $\omega-\mathbf{k}\langle\mathbf{u}\rangle>0$, the solution to Eq. (13) can be expressed in the form

$$\omega=\frac{\mathbf{k}\langle\mathbf{u}\rangle}{2(1-A)}\left\{2-A\pm\sqrt{A^2+4(1-A)\frac{\Omega^2}{(\mathbf{k}\langle\mathbf{u}\rangle)^2}}\right\}. \tag{14}$$

Relations (13) and (14) have been derived under the assumption that the dynamic viscosity of the fluidizing agent is rather low and that, consequently, the damping of collective vibrations is weak. From (14) one can obtain formulas for the phase velocity ω/k and the group velocity $d\omega/dk$ of propagation of collective vibrations through a fluidization bed of electrically charged particles (in a coordinate system referred to the fluid phase):

$$\begin{aligned}\frac{\omega}{k}&=\frac{\langle\mathbf{u}\rangle\mathbf{k}}{2(1-A)k}\left[2-A\pm\sqrt{A^2+4(1-A)\frac{\Omega^2}{(\mathbf{k}\langle\mathbf{u}\rangle)^2}}\right];\\ \frac{d\omega}{d\mathbf{k}}&=\frac{\langle\mathbf{u}\rangle}{2(1-A)}\left[2-A\pm\frac{A^2}{\sqrt{A^2+4(1-A)\frac{\Omega^2}{(\mathbf{k}\langle\mathbf{u}\rangle)^2}}}\right].\end{aligned} \tag{15}$$

It is worthwhile to examine the extreme cases of longwave and shortwave vibratory processes. Introducing the parameter

$$\varkappa=\frac{(\mathbf{k}\langle\mathbf{u}\rangle)^2A^2}{8(1-A)\Omega^2},$$

we have for longwave vibrations $\varkappa \ll 1$ when $A \ll 1$ and

$$\begin{aligned}\frac{\omega}{k}&=\frac{2-A}{2(1-A)}\cdot\frac{\mathbf{k}\langle\mathbf{u}\rangle}{k}\pm\frac{\Omega}{k\sqrt{1-A}}(1-\varkappa),\\ \frac{d\omega}{d\mathbf{k}}&=\frac{2-A}{2(1-A)}\langle\mathbf{u}\rangle\pm\frac{A^2\mathbf{k}\langle\mathbf{u}\rangle}{4(1-A)\Omega}\langle\mathbf{u}\rangle,\end{aligned} \tag{16}$$

while for shortwave vibrations $\varkappa \gg 1$ when $A \ll 1$ and

$$\begin{aligned}\frac{\omega}{k}&=\frac{2-A}{2(1-A)}\cdot\frac{\mathbf{k}\langle\mathbf{u}\rangle}{k}\pm\frac{A(\mathbf{k}\langle\mathbf{u}\rangle)}{2k(1-A)}\left(1+\frac{1}{\varkappa}\right),\\ \frac{d\omega}{d\mathbf{k}}&=\frac{2-A}{2(1-A)}\langle\mathbf{u}\rangle\pm\frac{A\langle\mathbf{u}\rangle}{2(1-A)}\left(1-\frac{1}{\varkappa}\right).\end{aligned} \tag{17}$$

When $q \to 0$, relation (14) becomes (11). When $\langle \mathbf{u} \rangle = 0$, relation (14) is identical to the expression for the Langmuir frequency [8] modified by a correction for the associated mass:

$$\omega = \pm \frac{\Omega}{\sqrt{1-A}}.$$

As an example, we will consider an air-fluidized bed of charged particles d = 1 mm in diameter and with a density $\rho_S = 2000$ kg/m^3. Let the charge of a single particle be $q \sim 10^{-11}$ C, which corresponds to a particle potential of about 100 V. Such potentials are attained, for instance, during the fluidization of dielectric particles in acrylic glass columns. Under these conditions the value of parameter A is of the order of 10^{-3} and the frequency Ω is of the order of 10 sec^{-1}. The phase velocity of the studied longwave vibrations can be determined from formula (16), if one considers that the wavelength is of the same order of magnitude as the characteristic linear dimension of the fluidization bed (say, 5 cm). In this case, $\omega/k \simeq 10$ cm/sec in a coordinate system referred to the solid phase.

A peculiar feature of the studied collective vibrations in a fluidization bed is that they are propagated only in the mean direction of the fluidizing agent flow. Only Langmuir vibrations at a frequency according to (18) can be propagated across the flow at $(\mathbf{k}\langle \mathbf{u} \rangle) = 0$ as well as at $\langle \mathbf{u} \rangle = 0$.

NOTATION

- D is the diffusivity, in the velocity space;
- m is the mass of a solid phase particle;
- g is the acceleration of free fall;
- d is the diameter of a solid phase particle in a fluidization bed;
- v* is the wave velocity;
- f* is the wave frequency;
- l is the characteristic linear dimension of a fluidization bed;
- $\mathbf{w}$ is the vector of mean velocity of fluidizing agent flow;
- $\mathbf{u}$ is the velocity of solid particles;
- $\langle \mathbf{u} \rangle$ is the mean velocity of solid particles;
- k is the wave vector;
- q is the electric charge of a single solid phase particle;
- μ is the dynamic viscosity of fluidizing agent;
- θ is the parameter simulating the hydrodynamic "temperature" of a fluidization bed [2];
- ρ_f is the density of fluidizing agent;
- ρ_S is the density of solid phase;
- I is the collision integral;
- ω is the vibration frequency;
- η is the concentration of solid phase particles;
- λ_1, λ_2 as in [2].

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